# Luminous-efficiency functions for point sources

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Luminous-efficiency functions for small fields were measured with flicker photometry and heterochromatic brightness-matching methods for the same two subjects. For the larger stimuli (22.9 and 11.5′), the luminous-efficiency functions differed between the two methods. Those obtained by flicker photometry showed a simple narrow shape, whereas those obtained by heterochromatic brightness matching showed increased sensitivity at short and long wavelengths. The difference, however, decreased as the stimulus was reduced to 2.3′, and the luminous-efficiency functions converged to curves that resemble the CIE $V(\lambda)$ or Judd's modification. It seems unnecessary to introduce a new luminous-efficiency function for point sources, which is fortunate for practical purposes.

## INTRODUCTION

A request for brightness-matching data and mathematical color-vision models was made by Kinney[1] in 1978, who was at that time the chairperson of CIE Technical Committee 1.4, Vision. Among other things, a need for defining a brightness luminous-efficiency function for point sources was emphasized. Although we have the photopic luminous-efficiency function $V(\lambda)$ and the color-matching function $\bar{y}_{10}(\lambda)$ to represent the spectral-sensitivity function of the visual system, they are valid only for stimuli of 2 or 10° of arc of visual angle. In everyday life we are often confronted with much smaller stimuli than 2°; traffic signals observed from far distances are just one example. Proper environmental design for such a situation cannot be possible without a luminous-efficiency function for small fields.

Some data are already available; listed in Table 1 are the authors[2–8] who measured foveal luminous-efficiency functions for stimuli smaller than 20′ of arc. Their experimetnal conditions are also listed. The stimulus presentation denoted as glancing refers to subjects glancing at the stimulus field for a short period while the stimulus was present; scanning refers to subjects scanning the test and the reference alternately. The wavelengths covered varied, depending on retinal illuminance levels and on which subjects were participating, and the maximum range is listed in the table. Two features can be extracted from these data about the luminous-efficiency function as the stimulus size is decreased. One is a sensitivity decrease at short wavelengths when the function was determined by flicker photometry and by the absolute threshold. The other is sensitivity decrease at both short and long wavelengths when the function was measured by heterochromatic brightness matching.

It is well known that a complication arises in defining the luminous-efficiency function for large stimuli, 2 and 10°, because different methods yield different functions. In particular, the heterochromatic brightness-matching method rises in sensitivity at short and long wavelengths compared with flicker photometry (CIE Technical Report).[9] These rises seem to disappear in small fields, and we can anticipate that the luminous-efficiency function for small fields may be freed from the complication encountered in a 2 or 10° field, at least at the central fovea.

It is noted in Table 1, however, that such anticipation cannot be confirmed because no author measured the function with both the flicker photometry and the heterochromatic brightness-matching methods for the same subjects and for the same conditions. In the present paper, we employ the two methods and measure the luminous-efficiency functions for small fields for the same two subjects and for the same retinal illuminance. We compare the functions to see if they reduce to a single and simple function.

## APPARATUS

A conventional two-channel Maxwellian optical-view system with a light source of a 500-W dc xenon-arc lamp was constructed to provide circular test stimuli of small sizes, 2.3, 11.5, 22.9, and 68.7 min of arc of visual angle. For flicker photometry, the fields from both channels were completely superimposed at the center of four red spots of 5′ diameter each, arranged at corners of a diamond with 2° separation diagonally. For heterochromatic brightness matching, an optical flat inserted in the reference optical channel was rotated slightly to shift the reference field toward the left so that the reference and the test stimulus were clearly separated from each other. The distance between the two varied depending on the sizes of stimulus, which was 23′ for 2.3 and 11.5′ stimulus size and 46′ for 22.9′ size when measured at the centers. A satisfactory separation was not achieved for 68.7′ stimulus with the optical flat, and the size was not investigated for heterochromatic brightness matching.

Magnetic shutters were inserted in both channels and operated electronically. They alternated the reference and the test stimulus in the flicker photometry with frequencies optimal for getting the minimum flicker. The frequencies differed, as was expected from the classical critical-flicker-frequency experiment of Hecht *et al.*,[10] and they were 6.3, 12.5, 16.7, and 20.0 Hz for 2.3, 11.5, 22.9, and 68.7′, respectively. The shutters also controlled the stimulus duration and the interstimulus interval between successive presentations. The

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**Table 1. Authors Who Investigated Spectal Sensitivity for Small Stimulus Size**

| Methods and Authors | Size | Retinal Illuminance (td) | Stimulus Presentation | Number of Subjects | Wavelengths Covered (nm) |
|---|---|---|---|---|---|
| Flicker photometry | | | | | |
| Abramov *et al.* (1977)[b] | 1.5° | 1200 | | | |
| | 5′ | 1200 | continuous | 6 | 450–660 |
| Heterochromatic-brightness matching | | | | | |
| Willmer *et al.* (1945)[c] | 2° | 100[a] | | 1 | 420–690 |
| | 20′ | 100[a] | | | |
| Thomson *et al.* (1947)[d] | 15′ | 50 | glancing | 2 | 460–650 |
| Thomson (1947)[e] | 15′ | 0.015–29 (six levels) | glancing | 1 | 480–650 |
| Bedford *et al.* (1958)[f] | 1° | 50 | | | |
| | 12′ | 500, 50, 7 | scanning | 4 | 400–700 |
| | 1.5′ | 11000, 1500, 150 | | | |
| Absolute threshold | | | | | |
| Sperling *et al.* (1957)[g] | 42′ | | 0.004 sec | 4 | |
| | 3′ | | 0.1 sec | 3 | 420–700 |
| Wald (1967)[h] | 62′ | | | 2 | |
| | 31.5′ | | 0.04 sec | 1 | 380–700 |
| | 15.5′ | | | 1 | |
| | 8.5′ | | | 2 | |

[a] Somewhat lower levels were used for short wavelengths.
[b] Ref. 2.
[c] Ref. 3.
[d] Ref. 4.
[e] Ref. 5.
[f] Ref. 6.
[g] Ref. 7.
[h] Ref. 8.

duration was fixed at 1 sec for both methods, and the stimulus was presented by an exposure button operated by the subject. The interstimulus interval was always adjusted longer than 2 sec regardless of the subject's choice of an earlier stimulus presentation. The short stimulus duration of 1 sec and the interstimulus interval longer than 2 sec were set to avoid the local adaptation that is common in small fields.

A 100-td white light with chromaticity coordinates $x = 0.336, y = 0.368$ was used as the reference light. The radiance of the test light was changed with a neutral-density wedge filter operated by subjects through a reversible motor. Interference filters from 398 through 697 nm with approximately 20-nm steps were used to provide wavelengths of test light. Their wavelengths were 398, 419, 440.5, 457, 481, 500, 523, 540, 560.5, 579, 602, 622, 640, 660, 679, and 697 nm. The 679-nm filter was replaced by 676.5-nm filter because the former was inadequate for 22.9′ test size in both subjects and 68.7′ test size in the subject YA.

A mouthpiece fixed the subject's head position. Two male graduate students of ages 24 (YA) and 26 (KY) years served as subjects. Their color vision was diagnosed as normal with the Ishihara plates.

## PROCEDURE

The subject was dark adapted for 5 min and then fixated at the center of four fixation dots. When he believed that the fixation was solid, he pressed the exposure button that delivered the stimulus for 1 sec. The subject adjusted the test-stimulus radiance for minimum flicker in the case of the flicker photometry and for equal brightness with the reference light in the case of heterochromatic brightness matching. He repeated the 1-sec exposures until he arrived at a satisfactory setting. He provided three such settings for each wavelength. The test wavelength was randomly chosen, and all 16 wavelengths were investigated in one session. Within one session both flicker photometry and heterochromatic brightness matching were carried out with a short intermission. Such a session was repeated 10 or 15 times so that the equality setting amounted to 30 or 45 times in total.

## RESULTS

Results of the flicker-photometry tests are shown in Fig. 1. The curves of subject KY are shifted downward by 1.5 log units. YA's points for 2.3′ (open circles) and KY's points for 2.3′ (open circles) and 11.5′ (open triangles) are the average of 45 measurements, whereas other points are the mean of 30 measurements. The ordinate shows the log-luminous efficiencies relative to 560 nm.

All curves show a relatively smooth shape, as expected for flicker photometry. They are close to one another at long wavelengths but show differences at short wavelengths. In the short-wavelength region the efficiency decreases as the field gets smaller, as was demonstrated by Abramov *et al.*[2] The decrease was gradual in YA, showing the narrowest function at 2.3′. In KY, however, the decrease was complete at 22.9′, and further narrowing did not take place.

Results of the heterochromatic brightness matching are shown in Fig. 2. YA's data points for 2.3′ (open circles) and KY's points for 2.3′ (open circles) and 11.5′ (open triangles) were obtained after 45 measurements; other points were ob-

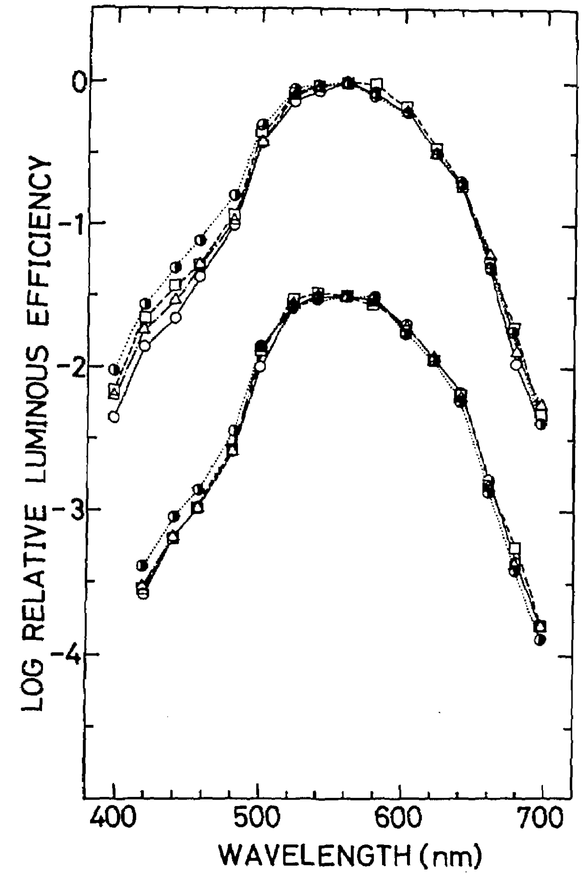


Fig. 1. Luminous-efficiency functions obtained by flicker photometry. Top: subject YA; bottom: subject KY. Stimulus size: ◐, 68.7′; □, 22.9′; △, 11.5′; ○, 2.3′.

tained after 30 measurements. Thick solid curves are theoretical and are based on a color-vision model that will be explained below.

The well-established features of luminous-efficiency function of 2 or 10° visual field still exist with 22.9′ (open squares) and 11.5′ (open triangles) in both subjects. That is, efficiency rises at short- and long-wavelength regions relative to the flicker curves. Such rises, however, disappear when the stimulus size is reduced to 2.3′ (open circles), in agreement with previous findings by Willmer *et al.*[3] and Bedford *et al.*[6]

## DISCUSSION

Our main aim was to investigate whether the luminous-efficiency functions for point sources differ as a function of the method of measurement. Before discussing that point, however, we analyze Figs. 1 and 2 separately.

We noted in Fig. 1 that spectral sensitivities by flicker photometry decreased at short wavelengths when the stimulus size was decreased to 2.3 min of arc of visual angle. Abramov *et al.*[2] also found a similar decrease with the same flicker photometry. The decrease, however, does not seem unique to flicker photometry. It was also found in the absolute threshold[7,8] and in the spectral thresholds for flicker perception.[11]

The sensitivity decrease cannot be explained by a density difference of macular pigmentation because the decrease occurs at the center of the fovea,[12,13] which would predict a sensitivity change in the opposite direction. All the abovementioned authors, therefore, were led to conclude the absence of blue cones from the center of the fovea, the phenomenon being known as the small-field tritanopia. Our results shown in Fig. 1 also support this.

The conclusion, however, leads us to a fundamental question of whether blue cones contribute to luminance. Luminance here is interpreted as luminous sensation that is specified by flicker photometry and by other equivalent methods, such as the minimally distinct border (MDB) method[14] and the temporal MDB method,[15] and is considered an output of the achromatic channel (sometimes called the luminance channel) in the visual-system model. In constructing the model, Vos *et al.*[16] assumed that the achromatic channel has inputs from all three kinds of cones (red, green, and blue) to produce luminance. Smith *et al*,[17] on the other hand, assumed that it has inputs from only two, red and green, and dropped blue cones from the achromatic channel. Tansley *et al.*[18] supported the latter model on the basis of their MDB experiment, and Eisner *et al.*[19] also favored the two-cone hypothesis. They conducted a flicker-photometry test between two wavelengths in the blue region and investigated the influence of the blue-adapting background. Since no effect from the blue background was found, they concluded that there is no contribution from blue cones to flicker photometry.

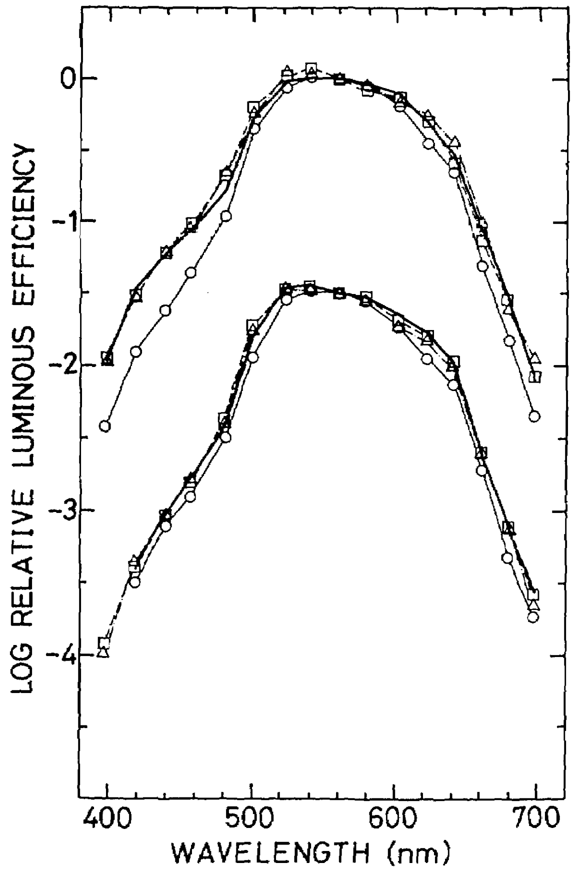


Fig. 2. Luminous-efficiency functions obtained by heterochromatic brightness matching. Top: subject YA, bottom: subject KY. Stimulus size: □, 22.9′; △, 11.5′; ○, 2.3′. Thick solid curves are theoretical.

Decrease of luminous efficiency at short wavelengths when the stimulus size is reduced is quite clear in our results shown in Fig. 1. The reduction of the blue-cone contribution seems most reasonable to assume, which appears to contradict the above findings. It is conceivable, however, that the effect of the adapting light on spectral sensitivity is difficult to observe in flicker photometry but is relatively easy to see in other measurements. It is also conceivable that the blue cone does not contribute to the MDB criterion that is related to visual acuity[20] but does to the flicker criterion.[21]

Luminous-efficiency functions of two large fields, 22.9 and 11.5′ of Fig. 2, which were determined with heterochromatic brightness matching, show familiar features that have been observed in the 2° field of the CIE TC 1.4 Technical Report.[9] The two curves of subject YA, in fact, overlap almost perfectly with the mean curve of the technical report. Sensitivity rises both in short and long wavelengths, yielding a broadened luminous-efficiency function. These rises disappear as the field decreases to 2.3′ to produce a normal narrow function.

It is understood through recent investigations[22–28] that the sensitivity rises in short- and long-wavelength regions are consequences of the chromatic channels' contribution to the brightness. Thus the disappearance of the sensitivity rises should indicate the reduction of the chromatic contribution when the test field is reduced to a small size of 2.3′. The effect of decreasing stimulus size was investigated for color naming,[29–32] hue perception,[33] wavelength discrimination,[34–35] and color mixture.[3] In our experiment we also asked subjects to specify subjectively hue and saturation of the test stimulus. All these studies indicate deterioration of chromatic sensation as the test field is made small, which supports the above analysis that the chromatic channels' contribution reduces.

As the luminous efficiency for flicker perception is determined solely by the achromatic channel, it is anticipated that the spectral-sensitivity curves of the heterochromatic brightness matching of Fig. 2 and the flicker photometry of Fig. 1 become alike when the test size is reduced. Figure 3 shows the difference between the two curves of the same test size relative to 560 nm. The difference is large at short and long wavelengths when the stimulus size is large [11.5′ (triangles) and 22.9′ (squares)] to exhibit a shape resembling the saturation-discrimination function determined relative to white and plotted against wavelength.[36,37] As the saturation-discrimination function is explained by the contribution from the chromatic channels,[38] the resemblance supports the interpretation that chromatic channels are still effective for brightness perception with these two large stimuli.

The difference becomes small, as anticipated, when the stimulus is reduced to 2.3′ (circles). The standard error of the mean after 45 determinations of luminous efficiency for this stimulus size is given by vertical bars at each wavelength investigated. In the case of subject YA (upper graph of Fig. 3), we can reject the hypothesis of no difference (significance level, 0.05) only at six wavelengths and may conclude that the two curves are almost the same. The contribution of the chromatic channels became ineffective with this small test stimulus. In subject KY (lower graph), the difference curve resembles two other curves in its overall shape, and there still seems to be chromatic contribution in heterochromatic brightness matching.

The difference in luminous efficiency between the two methods can be calculated on the basis of the color-vision model that involves the assumption that the luminous efficiency of heterochromatic brightness matching is determined both by the achromatic and chromatic channels. For example, Yaguchi *et al.*[28,39] expressed the luminous-efficiency function $V_b(\lambda)$ as follows:

$$V_b(\lambda) = \frac{1}{L_{e,\lambda}}. \tag{1}$$

$L_{e,\lambda}$ is the luminance needed to reach a certain criterion and is determined by the following nonlinear equation:

$$(\bar{a}_\lambda L_{e,\lambda})^2 + ts[(k_1\bar{c}_{1\lambda}L_{e,\lambda})^{2p} + (k_2\bar{c}_{2\lambda}L_{e,\lambda})^{2q}] = 1. \tag{2}$$

The first set of parentheses represents the contribution from the achromatic channel, and $\bar{a}_\lambda$ is equated to Judd's modification of $V(\lambda)$ as modified further by Vos.[40] The second bracket represents the contribution from the chromatic channels, and $\bar{c}_{1\lambda}$ and $\bar{c}_{2\lambda}$ are opponent-color responses of red versus green and yellow versus blue. The former is derived as a difference between responses of red and green cones, and the latter as a difference between red and green, and blue cones. We use the cone sensitivities proposed by Smith and Pokorny.[17,41] The exact contribution of each cone is spontaneously fixed by assigning certain wavelengths for unique hues of yellow and green. In the present calculation, wavelengths of 570 and 500 nm were employed, as given in a previous paper.[28] The nonlinear coefficients $p$ and $q$ were introduced to explain exactly the additivity failure of two colored lights for brightness,[28] and the equation reduces to that proposed by Guth *et al.*[23] if $p$ and $q$ are equated to unity. Actual values of $p$ and $q$ are, however, not critical to determine

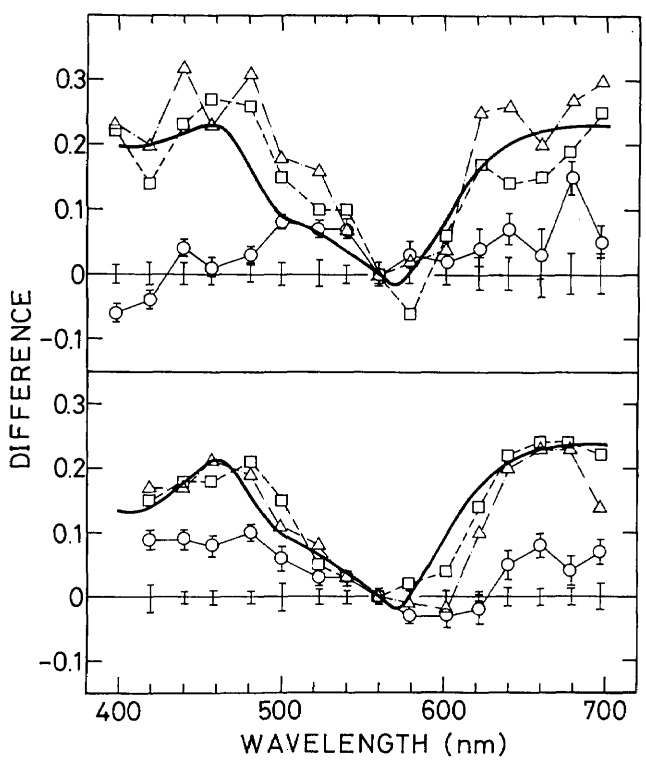


Fig. 3. Luminous-efficiency difference between heterochromatic brightness matching and flicker photometry for three stimulus sizes: □, 22.9′; △, 11.5′; ○, 2.3′. Standard errors of the mean for 2.3′ are given by vertical bars at open circles for heterochromatic brightness matching and at zero line for flicker photometry. Smooth thick solid curves are theoretical. Top: subject YA; bottom: subject KY.

the shape of $V_b(\lambda)$, and we use here the values $p = 0.64$ and $q = 0.36$ that were introduced before.[28]

$t$ and $s$ are temporal and spatial coefficients, respectively, and they are equal to unity at a moment. Then the only coefficients that remain to be determined in the equation are $k_1$ and $k_2$, and they were derived by the least-squares method so that the difference $\log V_b(\lambda) - \log \bar{a}_\lambda$ best fitted the experimental values of 22.9′ of Fig. 3. We obtained $k_1 = 0.96$ and $k_2 = 0.35$ for subject YA and $k_1 = 0.99$ and $k_2 = 0.21$ for KY. The theoretical difference between heterochromatic brightness matching and flicker photometry represented by $\log V_b(\lambda) - \log \bar{a}_\lambda$ is finally obtained, and it is shown by a smooth thick solid curve in Fig. 3. Log $V_b(\lambda)$ appeared in Fig. 2 as thick solid curves. These theoretical curves fit fairly well with the experimental results of 22.9′ in Figs. 2 and 3 to support validity of the form of Eq. (2) as well as the notion involved in the equation that chromatic channels also contribute to brightness perception.

From the results of the present experiment, we know that the contribution of the first set of parentheses on the left-hand side of Eq. (2) becomes smaller as the test size gets smaller and that it vanishes if flicker photometry is used. In other words, the temporal coefficient $t$ may take a value either of 1 or of 0, depending on the criterion employed. It is unity when the brightness perception is used and zero when flicker, MDB, or temporal MDB perception is used. The spatial coefficient $s$ may take a value between 1 and 0, depending on the stimulus size. In the 2.3′ curve of Fig. 1, $t = 0$ and $s = 0$, and in the 2.3′ curve of Fig. 2, $t = 1$ and $s$ is almost zero. In both cases $V_b(\lambda)$ converges to nearly $\bar{a}_\lambda$.

The foregoing discussion is summarized by Fig. 4, in which all the data points of 2.3′ of both subjects are replotted together with the CIE $V(\lambda)$ and Judd's modification. All the points come close to one another regardless of the methods employed. It is, therefore, suggested that either method would be appropriate to investigate the luminous-efficiency function for point sources at fovea. It is also suggested that the function can be represented by the CIE $V(\lambda)$ or Judd's modification fairly well. It is quite fortunate that we are freed from establishing yet another luminous-efficiency function for point sources. More experimental data are obviously needed, however, to decide which of the two, $V(\lambda)$ or Judd's modification, is finally chosen.

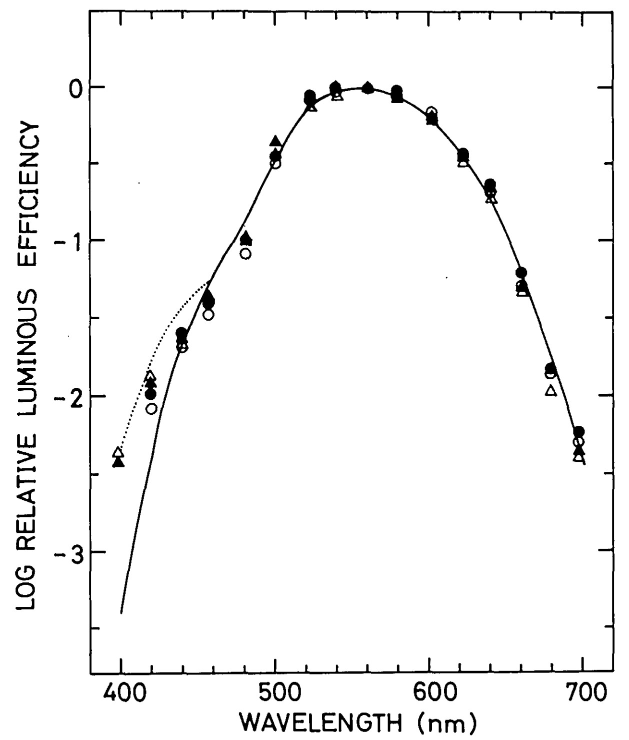


Fig. 4. Plots of luminous efficiencies for 2.3′ size, two subjects, and two methods. Triangles: subject YA, circles: subject KY. Open notations are for flicker photometry and filled for heterochromatic brightness matching. Solid curve, CIE V(λ); dotted curve, Judd's modification.

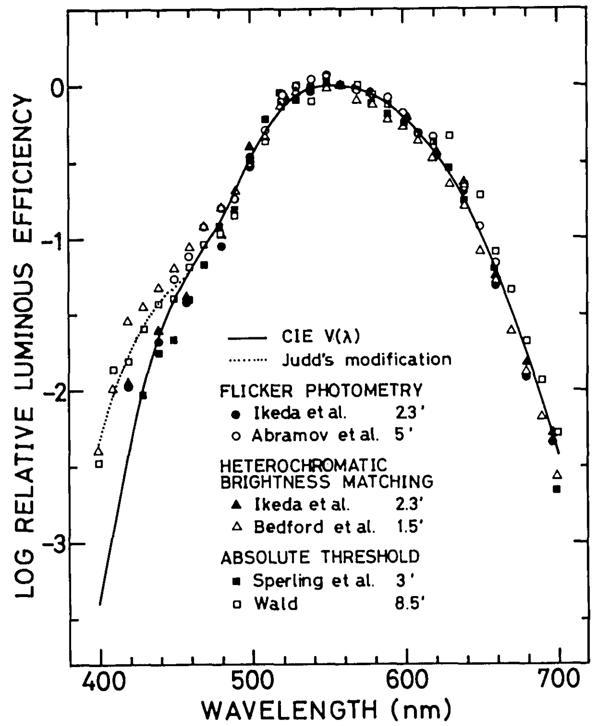


Fig. 5. Plots of spectral sensitivities of stimulus sizes smaller than 10′ of arc from various investigators. Authors' names and stimulus sizes are shown in the figure.

Spectral sensitivities for stimulus size smaller than 10′ obtained by various authors as they appear in Table 1 are plotted in Fig. 5 to compare them with one another and with our results. Values were read from graphs of the published papers and may not be accurate. It is nevertheless concluded that all points come relatively close to one another, confirming the convergence of all points to a narrow simple curve that may be represented by the existing $V(\lambda)$ or Judd's modification.

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